International Journal of Theoretical Physics, Vol. 6, No. 3 (1972), pp. 175–179

Point-like Hadrons from Colliding Beams

R. L. INGRAHAM

Research Center, New Mexico State University, Las Cruces, New Mexico 88001

Received: 14 *September* 1971

Abstract

We recall that a *frame-dependent* cut-off in field theory has predicted the phenomenon apparently observed in $e^+e \to$ 2 hadrons in colliding beams, namely that (hadronic or semi-hadronic) cross-sections $d\sigma/dt$ directly measured in the CM frame should be different (in general, larger) than those measured in the Lab frame at sufficiently high energies. A rough calculation shows quantitative agreement with the preliminary data.

The reaction

$$e^+ e \to \pi^+ \pi^- \tag{1}$$

directly measured in the CM frame at about $s \approx 4$ (GeV/c)^2, has revealed that the pions are 'point-like'.† CM frame multipion production cross-sections are also unexpectedly large. In the meager published results which we have seen, the words 'puzzle' and 'apparent contradiction' are already being used. Why is this 'puzzling'? Because protons—hadrons not expected to be too different dynamically from pions—were revealed as far from point-like in the similar *Lab frame experiment, at the equivalent Lab energy,*

$$\bar{p}p \to e^+ e \tag{2}$$

Happily, the time-reversed version of (2)

$$e^+ e \to \bar{p}p \tag{3}$$

was apparently also measured at Frascati, and is now being analyzed (Conversi, 1970; Panofsky, 1970).

Admittedly, these experiments are difficult, the analysis of the data only preliminary; in particular, (3) and (1) both suffer from problems of identifying the final two hadrons and separating off final state interactions. Never-

† See *Physics Today* (December 1970), pp. 17 and 19; Conversi (1970); and Panofsky (1970).

theless, for the purposes of this article, we shall accept the preliminary conclusions quoted above.†

The purpose of this present work is to point out that there is a theory, in the published literature since 1962, whose main characteristic is precisely this sort of phenomenon. We list here a few of the more recent works, which adequately summarize the formalism: Ingraham (1964b, 1969a, b); Bailey & Ingraham (1966); and Yodzis & Ingraham (1970). In particular, the prediction of unequal cross-sections $d\sigma/dt$ for the same reaction *measured* at equivalent energies and angles (same s and t) in the CM and Lab frames (the former in general larger) was explicitly made many times (see, for example, Ingraham, 1964a, 1967, 1969a, b; and Bailey & Ingraham, 1966). This theory has received little attention; any unambiguous test of it had in fact necessarily to wait on the comparison of the same or similar reactions performed in the CM and Lab frames at sufficiently high energies.‡ We predict on the basis of this theory that the process (3) will also show a 'point' proton (that is, form facter F_{1p} of the order of unity) or at least one more point-like than that given by the Lab frame experiment (2). It is difficult to say more because of the dynamical uncertainties, especially at these 'low' energies. If s were large relative to our cut-off λ^{-2}, estimated to be $\geqslant(2\ \mathrm{GeV}/c)^2$, one could predict a point proton from process (3) with more confidence (see below).

Needless to say, if (3) does reveal a point proton, or even a proton form factor definitely different from that yielded by experiment (2), a crisis of the first magnitude would be produced, and the cry 'breakdown of relativistic invariance!' would be heard in the land. For, regardless of their manifold differences in dynamical mechanism, all the currently popular theories subscribe to the same relativistic transformation law, and hence would necessarily predict equal $d\sigma/dt$ for (2) and (3) at the same s and t.§

However, this phenomenon would not prove a violation of relativistic invariance correctly understood (i.e., the relativity principle, the exact equivalence of all inertial frames) but only of the extra postulate, tacitly made in scattering theory, that a cross-section is independent of the frame in which it is measured. Namely, if p_i, $i = 1, 2, \ldots$, are the incoming and outgoing momenta, and $n(\mathscr{L})$ the *frame vector* (unit time-like vector along frame $\mathscr{L}$'s time axis), then present-day particle theory allows $d\sigma/dt$ to be a function only of the invariants $p_i \cdot p_j$, while frame-dependent theory allows dependence on the $p_i \cdot n(\mathscr{L})$ also. Both satisfy commonly accepted statements of the relativity principle; for example, 'If equivalent observers (inertial

† The preliminary data used in this paper have been confirmed by further experimental work and analysis: G. Salvini, Talk JM3 of the April Meeting of the APS in Washington, D.C. and private communication.

‡ Originally, we thought that there should be one, universal cut-off λ for all kinds of particles. However, the assumption of at least two cut-offs, one for pure QED and one for graphs in which hadrons participate, with $\lambda_{\mathrm{QED}} \ll \lambda_{\mathrm{H}}$, now seems more plausible, and is our hypothesis in this article. The size $\lambda_{\mathrm{H}} \approx 1 \times 10^{-14}$ cm comes from rough calculations of hadron EM mass shifts, hadron total cross-sections, etc.

§ Granting always that T-invariance is accepted. Proton mass is neglected relative to s.

frames) do identical experiments (same set of numbers p_i), they must get identical numbers', or 'No inertial frame can be distinguished from any other by experiments performed within the frame'. The relativity principle enforces a Poincaré group transformation law of quantum fields (or amplitudes) in which the vectors $n(\mathscr{L})$ transform among themselves as well as the point x (or momenta p_i). Although this, of course, is the theoretical point of prime importance, far transcending any dynamical peculiarities of (2) or (3), it will not be argued here, since it has been adequately covered elsewhere [especially by Ingraham (1969a, b)].

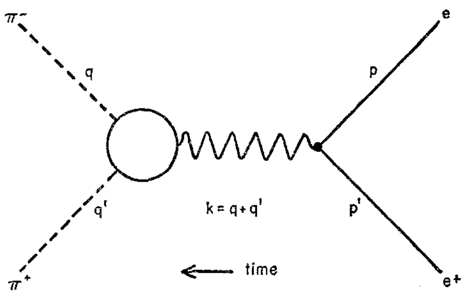

Figure 1.—Single photon exchange graph for the reaction $e^+e \to \pi^+\pi^-$.

Now we show how the theory alluded to above gives these experimental results even semi-quantitatively already in the simplest approximation. It is a field theory with a cut-off, where the frame-dependence (allowed on general grounds as we have said) enters in this particular case via the random underlying space-time responsible for the cut-off.

Assuming that the graph of Fig. 1 is sufficient for process (1) at these energies, the cut-off theory's graph rules† give an *effective* pion EM form factor

$$F_{\pi\text{eff}} = |g(k)|^2 F_{\pi\text{dyn}} \tag{4}$$

where the kinematical form factor $|g(k)|^2$ is that attached to every internal line and $F_{\pi\text{dyn}}$ is the *dynamical* form factor (vertex) due to the strong interactions in the blob shown in the graph. The F's are functions of the invariants formable from the vectors q and q' (or q and k, etc.) *and* $n(\mathscr{L})$; these arguments will be suppressed, because we shall not need them.

$|g(k)|^2$, however, is a function of k and $n(\mathscr{L})$ only, in the form $x \equiv k_\perp{}^2\lambda^2$, where

$$k_\perp{}^2 \equiv k^2 + [k.n(\mathscr{L})]^2 \equiv \text{3-momentum } \mathbf{k}^2 \text{ in frame } \mathscr{L} \tag{5}$$

and λ is the hadronic cut-off length, taken here as $\lambda \approx 1 \times 10^{-14}$ cm from other work. Its functional form has been determined from random space-

† See, for example, Bailey & Ingraham (1966, p. 1291). But unlike that paper we now believe it correct to define the cross-section by the usual formula in terms of the modified amplitude M'_{fi} in which the *external line* kinematical form factors $g(k)$ and $g^*(k)$ have been divided out.

time geometry alone: it starts off like an exponential in x, and asymptotically ($x \gg 1$) goes like $(2\pi x)^{-1}$. It equals unity at $x = 0$ for a massless propagator, as in our graph.

Now for (1), $\mathscr{L} = \text{CM}$ frame, so that $k_\perp{}^2 = 0$ from (5), and hence $|g(k)|^2 = 1$. The experiments show $F_{\pi\text{eff}} =$ order of 1, more precisely $\approx 1/\sqrt{2}$ (Conversi, 1970; Panofsky, 1970). So we conclude

$$F_{\pi\text{dyn}} \approx 1/\sqrt{2}, \qquad s = (2{\cdot}2\ \text{GeV/c})^2, \qquad \mathscr{L} = \text{CM} \tag{6}$$

This is reasonable, because for this cut-off theory simple perturbation theory has shown that the dynamical form factors, in general, go back to their threshold values (here unity) as any of $q_\perp{}^2\lambda^2$, $q_\perp{}'^2\lambda^2$, or $k_\perp{}^2\lambda^2 \to \infty$, as a result of the cut-off (Ingraham, unpublished work; Genolio, 1963); and there is no reason to doubt this for the rigorous theory.

At this point we get the prediction that (3) will reveal

$$F_{1p\,\text{eff}} = 0(1), \qquad s = (2{\cdot}2\ \text{GeV/c})^2, \qquad \mathscr{L} = \text{CM} \tag{7}$$

or at least $> \frac{1}{10}$, because $\mathscr{L}$ is again the CM frame and one expects the dynamical form factors of π and p to be roughly the same.

Now we come to the Lab frame experiment (2), which showed (Conversi, 1970; Panofsky, 1970) $F_{1p\,\text{eff}} \leqslant \frac{1}{10}$. Assuming that a graph like Fig. 1 suffices, one gets equation (4) with $F_\pi \to F_{1p}$. But now, $\mathscr{L} = \text{Lab}$, so that $k_\perp{}^2 \equiv p_{\text{Lab}}^2 \approx (2{\cdot}5\ \text{GeV/c})^2$ for $s = (2{\cdot}2\ \text{GeV/c})^2$. Thus $x \equiv k_\perp{}^2\lambda^2 \approx 1{\cdot}6$, and a look at its graph shows $|g(k)|^2 \approx 0{\cdot}14$ at this value (the asymptotic formula gives already $\approx \frac{1}{10}$). On the other hand, $F_{1p\,\text{dyn}}$ is some function of k^2, $k_\perp{}^2$, $p_\perp{}^2$, $p_\perp'^2$ which we cannot fix unambiguously via the CM frame measurement of F_π [since although $k^2 = -s = -(2.2)^2$ is the same, $k_\perp{}^2$, $p_\perp{}^2$, and $p_\perp'^2$ are not†], but for which we will nevertheless guess

$$F_{1p\,\text{dyn}} \approx F_{\pi\text{dyn}} \approx 1/\sqrt{2}.$$

Hence

$$F_{1p\,\text{eff}} = |g(k)|^2 F_{1p\,\text{dyn}} \approx 0{\cdot}10, \qquad s = (2{\cdot}2\,\text{GeV/c})^2, \qquad \mathscr{L} = \text{Lab} \tag{8}$$

to compare with the experimental $F_{1p\,\text{eff}} \leqslant \frac{1}{10}$.

Concluding Remarks

1. A similar phenomenon should be observed in pure hadronic reactions, e.g., $pp \to pp$.‡ This could in principle be done at the NAL and CERN

† In the present case p, p' are the momenta of the incoming $p\bar{p}$ pair. Note that in any frame $k_\perp{}^2, p_\perp{}^2, p_\perp'^2$ are functions of the sole invariant k^2, but these functions vary with the frame. E.g., for $\mathscr{L} = \text{Lab}$, $p_\perp{}^2 = 0$, $k_\perp{}^2 = p_\perp'^2 = p_{\text{Lab}}^2 \approx (s/2M)^2$, $M \equiv$ proton mass.

‡ As reported in *Physics Today* (August 1971), p. 17 and Holder (1971), initial results show that the slope parameter b, where

$$\frac{d\sigma}{dt} = A\exp(bt)$$

as measured in the ISR at $p_{\text{CM}} \approx 25$ GeV/c is anomalously small compared to expectations motivated by Lab frame experiments at much lower values of s. This is in accord with our general expectation that the CM $d\sigma/dt$ should be larger than the Lab $d\sigma/dt$ at any (high) s and t.

colliding beams (ISR) accelerators; one must arrange the experiments to give an overlap in s and t. If we accept the Frascati data as good support for hadronic cut-off $\lambda \approx 1 \times 10^{-14}$ cm [0·5 $(\mathrm{GeV}/c)^{-1}$], then the disparity in the two cross-sections $d\sigma/dt$, measured at the same s and t, could be very large at the energies attainable there. The exact magnitude of the ratio depends of course on the dynamics, for which probably no single graph like Fig. 1 will suffice.

2. 'Inclusive' process cross-sections like

$$ep \rightarrow eX \tag{9}$$

$X =$ anything, have also been observed to be large† ('point-like proton') even though measured in the Lab frame. Although we are far from positive, the mechanism here is probably essentially different from that of the two-body process (1). On our theory, even though each separate cross-section, with X a definite set of hadrons in (9), should be falling in t because of the Lab frame kinematical form factor on the photon line (among others); nevertheless, the many inelastic channels, whose number increases rapidly with s, add up to keep the cross-section up.

Thus we would guess that also $e^+e \rightarrow$ any number of hadrons would remain large with s even when measured in the Lab frame, although it would be less than the value measured in the CM.

Acknowledgements

I thank Richard Taylor and W. K. H. Panofsky for conversations, and M. Conversi, B. Touschek and C. Bernardini for data and illuminating letters.

References

Bailey, D. and Ingraham, R. L. (1966). *Physical Review*, **152**, 1290.
Conversi, M. (1970). Nota Interna n. 281/Rev., Instituto di Fisica, Università di Roma.
Genolio, R. (1963). Doctoral thesis. New Mexico State University.
Holder, M., Radermacher, E., Staude, A., Barbiellini, G., Darriulat, P., Hansroul, M., Orito, S., Palazzi, P., Santroni, A., Strolin, P., Tittel, K., Pilcher, J., Rubbia, C., de Zorzi, G., Macri, M., Sette, G., Grosso-Pilcher, C., Fainberg, A. and Maderni, G. (1971). *Physics Review Letters*, **35B**, 355.
Ingraham, R. L. (1964a). *Nuovo cimento*, **32**, 323.
Ingraham, R. L. (1964b). *Nuovo cimento*, **34**, 182.
Ingraham, R. L. (1967). *Proceedings of the International Symposium on Nonlocal Quantum Field Theory*, p. 81. Dubna, U.S.S.R.
Ingraham, R. L. (1969a). *International Journal of Theoretical Physics*, Vol. 2, No. 1, p. 83.
Ingraham, R. L. (1969b). *Lectures in Theoretical Physics*, p. 951. University of Colorado Press, Boulder.
Panofsky, W. (1970) SLAC-PUB-816.
Yodzis, P. and Ingraham, R. L. (1970). *Physical Review*, **D2**, 1493.

† See, for example, F. Gilman's article in the *Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, September 1969*, edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Daresbury, England, 1969).